Math 116
Spring 2017

Study Guide for Exam 2

1. Which of the following describes the derivative function $f'(x)$ of a quadratic function $f(x)$?

(A) Cubic **(B)** Quadratic **(C)** Linear

(D) Constant **(E)** None of these

2. Find the derivative of the function $f(x) = x^3 - 12x^2 + \dfrac{x}{2} + 1$.

(A) $f'(x) = 3x^2 - 24x + \dfrac{1}{2}$ **(B)** $f'(x) = \dfrac{x^4}{4} - 4x^3 + \dfrac{x^2}{3} + x$ **(C)** $f'(x) = 6x - 24$

(D) $f'(x) = \dfrac{x^2}{3} - 6x + 1$ **(E)** None of these

3. Find the derivative of the function $g(x) = -10x^{-2} + \dfrac{6}{x^7}$.

(A) $g'(x) = \dfrac{10}{x} - \dfrac{1}{x^6}$ **(B)** $g'(x) = -\dfrac{20}{x} + \dfrac{42}{x^6}$ **(C)** $g'(x) = 20x - 42x^6$

(D) $g'(x) = \dfrac{20}{x^3} - \dfrac{42}{x^8}$ **(E)** None of these

4. Find the derivative of the function $h(x) = \dfrac{-5}{\sqrt[4]{x}}$.

(A) $h'(x) = -\dfrac{5}{3\sqrt[3]{x}}$ **(B)** $h'(x) = \dfrac{20\sqrt[4]{x^3}}{3}$ **(C)** $h'(x) = \dfrac{5}{4\sqrt[4]{x^5}}$

(D) $h'(x) = -\dfrac{20}{\sqrt[4]{x^3}}$ **(E)** None of these

5. Explain the relationship between the slope and the derivative of $f(x)$ at $x = a$. Choose the correct answer below.

(A) The derivative of $f(x)$ at $x = a$ describes the rate of change for the slope of the function at $x = a$.

(B) The derivative of $f(x)$ at $x = a$ equals the slope of the function at $x = a$.

(C) The slope of the function at $x = a$ describes the rate of change for the derivative of $f(x)$ at $x = a$.

(D) The derivative of $f(x)$ at $x = a$ is unrelated to the slope of the function at $x = a$.

(E) None of these.

6. Assume that a demand equation is given by $p = 120 - \frac{1}{100}q$. Find the marginal revenue for the production level $q = 1400$ units.

   The marginal revenue at 1400 units is

   (A) between 0 and 25 **(B)** between 25 and 50 **(C)** between 50 and 75

   (D) between 75 and 100 **(E)** between 100 and 125

7. Assume that a demand equation is given by $q = 9000 - 100p$. Find the marginal revenue for the production level $q = 3000$ units.

   The marginal revenue at 3000 units is

   (A) between 15 and 25 **(B)** between 25 and 35 **(C)** between 35 and 45

   (D) between 45 and 55 **(E)** between 55 and 65

8. Explain the concept of marginal cost. How does it relate to cost? How is it found? Answer parts **(i)** and **(ii)**

(i) How does the marginal cost relate to cost?

(A) Marginal cost refers to the rate of change of cost.

(B) Cost refers to the rate of change of marginal cost.

(C) Marginal cost is the same as cost.

(ii) How is the marginal cost found?

(A) Marginal cost is found by taking the derivative of cost.

(B) Marginal cost is found by taking the antiderivative of cost.

(C) Marginal cost is the same as cost.

9. Find the derivative of the function $f(x) = (5x^2 + 2)(5x - 2)$.

(A) $f'(x) = 75x^2 - 20x + 10$ **(B)** $f'(x) = 50x$ **(C)** $f'(x) = \dfrac{25x^2 - 20x - 10}{(5x-2)^2}$

(D) $f'(x) = 75x^2$ **(E)** None of these

10. Find the derivative of the function $g(x) = \dfrac{x^2 - 4x + 1}{x^2 + 7}$.

(A) $g'(x) = \dfrac{4x^3 - 8x^2 + 16x - 28}{(x^2+7)^2}$ **(B)** $g'(x) = \dfrac{x-2}{x}$ **(C)** $g'(x) = \dfrac{-8x^2 + 16x - 28}{(x^2+7)^2}$

(D) $g'(x) = \dfrac{4x^2 + 12x - 28}{(x^2+7)^2}$ **(E)** None of these

11. Find the derivative of the function $h(x) = \dfrac{(4x^2 + 4)(5x + 2)}{9x - 5}$.

(A) $h'(x) = \dfrac{40x}{9}$ **(B)** $h'(x) = \dfrac{60x^2 + 16x + 20}{9}$

(C) $h'(x) = \dfrac{360x^3 - 228x^2 - 80x - 172}{(9x-5)^2}$ **(D)** $h'(x) = \dfrac{-180x^3 + 228x^2 - 380x - 72}{(9x-5)^2}$

(E) None of these

12. Suppose that $f(x)$ and $g(x)$ are differentiable functions such that $f(2) = 1$, $f'(2) = 6$, $g(2) = 4$, and $g'(2) = 9$. Find $h'(2)$ when $h(x) = f(x) \cdot g(x)$.

(A) 15 **(B)** 33 **(C)** 54 **(D)** 58 **(E)** None of these

13. Suppose that $f(x)$ and $g(x)$ are differentiable functions such that $f(-3) = 8$, $f'(-3) = -4$, $g(-3) = -2$, and $g'(-3) = 5$. Find $h'(-3)$ when $h(x) = \dfrac{f(x)}{g(x)}$.

(A) -8 **(B)** $-\dfrac{4}{5}$ **(C)** 8 **(D)** 12 **(E)** None of these

14. The total cost (in hundreds of dollars) to produce x units of a product is

$$C(x) = \frac{2x-1}{6x+7}.$$

Answer parts **(i)** through **(iii)**

(i) Find $C'(x)$, the marginal cost function.

(A) $C'(x) = \dfrac{-20}{(6x+7)^2}$ **(B)** $C'(x) = \dfrac{20}{(6x+7)^2}$ **(C)** $C'(x) = \dfrac{-24x-8}{(6x+7)^2}$

(D) $C'(x) = \dfrac{24x+8}{(6x+7)^2}$ **(E)** $C'(x) = \dfrac{2}{6}$

(ii) Find $\overline{C}'(x)$, the marginal average cost function.

(A) $\overline{C}'(x) = \dfrac{12x^2+14x+7}{(6x^2+7x)^2}$ **(B)** $\overline{C}'(x) = \dfrac{-12x^2-14x-7}{(6x^2+7x)^2}$

(C) $\overline{C}'(x) = \dfrac{12x^2-12x-7}{(6x^2+7x)^2}$ **(D)** $\overline{C}'(x) = \dfrac{-12x^2+12x+7}{(6x^2+7x)^2}$

(E) $\overline{C}'(x) = \dfrac{20}{(6x+7x)^2}$

(iii) Find the marginal average cost for 10 units. The marginal average cost is

(A) between -\$0.60 and -\$0.50 per unit **(B)** between -\$0.50 and -\$0.40 per unit

(C) between -\$0.40 and -\$0.30 per unit **(D)** between -\$0.30 and -\$0.20 per unit

(E) between -\$0.20 and -\$0.10 per unit

15. A company that makes bicycles has determined that a new employee can assemble

$$M(d) = \frac{104d^2}{5d^2 + 6}$$

bicycles per day after d days of on-the-job training. Answer parts **(i)** and **(ii)**

(i) Find the rate of change function for the number of bicycles assembled with respect to time.

(A) $M'(d) = \dfrac{104d^3}{5d^3 + 2d}$ **(B)** $M'(d) = \dfrac{1248d}{(5d^2 + 6)^2}$ **(C)** $M'(d) = \dfrac{104}{5}$

(D) $M'(d) = \dfrac{2080d^3 + 1248d}{(5d^2 + 6)^2}$ **(E)** None of these

(ii) Find and interpret $M'(2)$.
Choose the correct interpretation.

(A) After 2 days of on the job training, the new employee can assemble about 3.7 more bikes than after 1 day of on the job training.

(B) After 2 days on the job, the new employee can assemble 3.7 bikes/day.

(C) After 2 days of on the job training, the new employee can assemble 3.7 bikes/day.

(D) After 2 days on the job, the new employee needs to learn to assemble 3.7 more bikes/day.

(E) None of these.

16. Find the derivative of the function $g(x) = (x^2 + 3x)^7$.

(A) $g'(x) = 7(2x+3)^6$

(B) $g'(x) = 7(x^2+3x)^6(2x+3)$

(C) $g'(x) = 7x^6(2x+3)$

(D) $g'(x) = 7(2x+3)^6(x^2+3x)$

(E) None of these

17. Find the derivative of the function $f(x) = 47(9x^3 - 8)^{3/2}$.

(A) $f'(x) = \dfrac{3807}{2}x^2(9x^3-8)^{1/2}$

(B) $f'(x) = \dfrac{141}{3}(27x^2)^{1/2}$

(C) $f'(x) = \dfrac{141}{3}(9x^3-8)^{1/2} + 27x^2$

(D) $f'(x) = \dfrac{3807}{2}x^{7/2}$

(E) None of these

18. Find the derivative of the function $f(x) = (5x+1)^5(4x+1)^{-2}$.

(A) $f'(x) = \dfrac{(5x+1)^4(-160x^2-72x+17)}{(4x+1)^2}$

(B) $f'(x) = -\dfrac{200(5x+1)^4}{(4x+1)^3}$

(C) $f'(x) = \dfrac{(5x+1)^4(10x+3)}{(4x+1)^3}$

(D) $f'(x) = \dfrac{(5x+1)^4(60x+17)}{(4x+1)^3}$

(E) None of these

19. Consider the following table of values of the functions f and g and their derivatives at various points.

x	1	2	3	4
$f(x)$	3	4	1	2
$f'(x)$	-4	-6	-9	-8
$g(x)$	3	4	2	1
$g'(x)$	5/9	4/9	8/9	1

Answer parts **(i)** through **(iv)**

(i) Find $\frac{d}{dx}[g(x) + f(x)]$ at $x = 4$.

The value of $\frac{d}{dx}[g(x) + f(x)]$ at $x = 4$ is

(A) less than -6 **(B)** between -6 and -2 **(C)** between -2 and 2

(D) between 2 and 6 **(E)** more than 6

(ii) Find $\frac{d}{dx}[f(x) \cdot g(x)]$ at $x = 1$.

The value of $\frac{d}{dx}[f(x) \cdot g(x)]$ at $x = 1$ is

(A) less than -6 **(B)** between -6 and -2 **(C)** between -2 and 2

(D) between 2 and 6 **(E)** more than 6

(iii) Find $\frac{d}{dx}\left[\frac{f(x)}{x^2}\right]$ at $x = 2$.

The value of $\frac{d}{dx}\left[\frac{f(x)}{x^2}\right]$ at $x = 2$ is

(A) less than -6 **(B)** between -6 and -2 **(C)** between -2 and 2

(D) between 2 and 6 **(E)** more than 6

(iv) Find $\frac{d}{dx}[g(f(x))]$ at $x = 3$.

The value of $\frac{d}{dx}[g(f(x))]$ at $x = 3$ is

(A) less than -6 **(B)** between -6 and -2 **(C)** between -2 and 2

(D) between 2 and 6 **(E)** more than 6

20. Consider the following table of values of the function f and its derivative:

x	2	3	6
$f(x)$	19	18	11
$f'(x)$	$-\frac{1}{2}$	-1	-5

Find $\dfrac{d}{dx}[f(2x)]$ at $x = 3$.

(A) -10 **(B)** -5 **(C)** -2 **(D)** $-\frac{1}{2}$ **(E)** 11

21. Find the equation of the tangent line to the graph of

$$f(x) = \sqrt{x^2 + 24}$$

at $x = 5$.

(A) $y = \frac{5}{7}(x-5)+7$ **(B)** $y = \frac{5}{7}(x-7)+5$ **(C)** $y = \frac{1}{14}(x-5)+7$

(D) $y = \frac{1}{14}(x-7)+5$ **(E)** None of these

22. Suppose the demand for a certain brand of a product is given by

$$D(p) = -\frac{p^2}{104} + 450.$$

where p is the price in dollars. If the price, in terms of the cost c, is expressed as $p(c) = 2c - 12$, find the demand function in terms of the cost.

(A) $D(c) = \dfrac{11664 - c^2}{26}$ **(B)** $D(c) = \dfrac{11664 + 12c - c^2}{26}$ **(C)** $D(c) = \left(450 - \dfrac{p^2}{104}\right)(2c - 12)$

(D) $D(c) = \dfrac{46806 - c}{52}$ **(E)** None of these

23. Find the derivative of the function $g(x) = 2e^{4x+1}$.

(A) $g'(x) = 2e^{4x+1}$ **(B)** $g'(x) = 8e^{4x+1}$ **(C)** $g'(x) = 2e^4$

(D) $g'(x) = 2(\ln 4)e^{4x+1}$ **(E)** None of these

24. Find the derivative of the function $h(x) = (2x^2 - 4x + 4)e^{-4x}$.

(A) $h'(x) = (-4x + 1)e^{-4x}$ **(B)** $h'(x) = (-16x^2 + 16x)e^{-3x}$

(C) $h'(x) = (-16x + 16)e^{-4x}$ **(D)** $h'(x) = (-8x^2 + 20x - 20)e^{-4x}$

(E) None of these

25. Find the derivative of the function $f(x) = \dfrac{8e^{4x}}{5x-2}$.

(A) $f'(x) = \dfrac{32e^{4x}}{5}$ **(B)** $f'(x) = \dfrac{8e^{4x}(20x-13)}{(5x-2)^2}$ **(C)** $f'(x) = \dfrac{8e^4}{5}$

(D) $f'(x) = \dfrac{8e^{4x}}{5}$ **(E)** $f'(x) = \dfrac{8e^{4x}(13-20x)}{(5x-2)^2}$

26. Find the derivative of the function $g(x) = 4^{6x+1}$.

(A) $g'(x) = 6(\ln 4)4^{6x+1}$ **(B)** $g'(x) = (\ln 4)4^{6x+1}$ **(C)** $g'(x) = 6 \cdot 4^{6x+1}$

(D) $g'(x) = 4^6$ **(E)** $g'(x) = (6x+1)4^{6x}$

27. Find the equation of the tangent line to $f(x) = e^{2x} + 3$ at $x = 0$.

(A) $y = 4x + 2$ **(B)** $y = 5x + 2$ **(C)** $y = 2xe^{2x} + 4$ **(D)** $y = 2x + 4$ **(E)** None of these

28. The sales of a new high-tech item are given by

$$S(t) = 9400 - 9000e^{-0.4t},$$

where t represents time in years. Find the rate of change of sales after 5 years. (Round to one decimal place as needed.)

(A) 3.3 **(B)** 487.2 **(C)** 8182.0 **(D)** 9887.2 **(E)** None of these

29. Using data in a car magazine, we constructed the mathematical model

$$y = 100e^{-0.08044t}$$

for the percent of cars of a certain type still on the road after t years. Answer parts **(i)** and **(ii)**

(i) Find the percent of cars on the road after 5 years.
The percent of cars is

(A) less than 50% **(B)** between 50% and 60% **(C)** between 60% and 70%

(D) between 70% and 80% **(E)** more than 80%

(ii) Find the rate of change of the percent of cars still on the road after 5 years.
The rate of change is

(A) between -10% and -8% per year **(B)** between -8% and -6% per year

(C) between -6% and -4% per year **(D)** between -4% and -2% per year

(E) between -2% and 0% per year

30. Find the derivative of the function $f(x) = \ln(5 - 4x)$.

(A) $f'(x) = -4\ln(5 - 4x)$ **(B)** $f'(x) = \dfrac{1}{5 - 4x}$ **(C)** $f'(x) = \dfrac{-4}{5 - 4x}$

(D) $f'(x) = \dfrac{1}{\ln(5 - 4x)}$ **(E)** $f'(x) = \dfrac{-4}{x}$

31. Find the derivative of the function $g(x) = \dfrac{4\ln(4x)}{4 + 5x}$.

(A) $g'(x) = \dfrac{16 + 20x - 20x\ln(4x)}{x(4 + 5x)^2}$ **(B)** $g'(x) = \dfrac{1}{5x}$ **(C)** $g'(x) = \dfrac{4}{5x}$

(D) $g'(x) = \dfrac{4 + 5x - 20x\ln(4x)}{x(4 + 5x)^2}$ **(E)** None of these

32. Find the derivative of the function $f(x) = \log(6x)$.

(A) $f'(x) = \dfrac{1}{x}$ **(B)** $f'(x) = \dfrac{1}{6x}$ **(C)** $f'(x) = \dfrac{1}{x\ln(10)}$

(D) $f'(x) = \dfrac{\ln(10)}{x}$ **(E)** $f'(x) = \dfrac{\ln(10)}{6x}$

33. The cost function (in dollars) for q units of a certain item is $C(q) = 102q + 91$. The revenue function (also in dollars) for the same item is

$$R(q) = 102q + \frac{54q}{\ln(q)}.$$

Answer parts **(i)** through **(iii)**

(i) Find the marginal cost function ($MC(q)$).

(A) $MC(q) = 102$ **(B)** $MC(q) = q$ **(C)** $MC(q) = 91$

(D) $MC(q) = 102q + 91$ **(E)** None of these

(ii) Find the marginal revenue function ($MR(q)$).

(iii) Find the profit function.

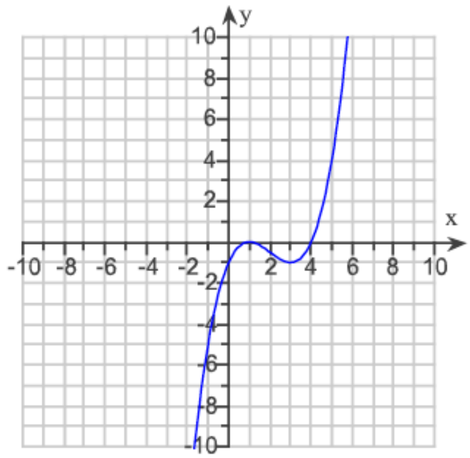
(iv) Find the marginal profit when 8 units are sold.
The marginal profit is

(A) less than \$9 per unit **(B)** between \$9 and \$10 per unit

(C) between \$10 and \$11 per unit **(D)** between \$11 and \$12 per unit

(E) more than \$12 per unit

34. Determine the critical number(s) of the function graphed below.

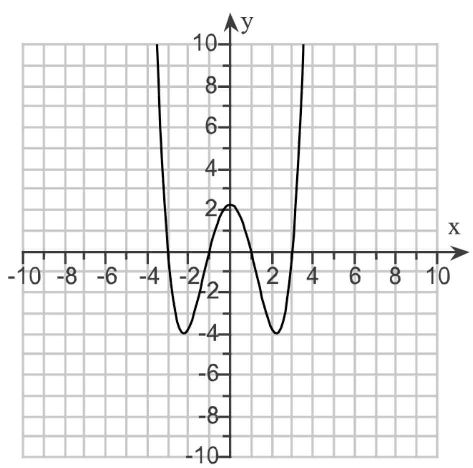


(A) $x = 0$ **(B)** $x = -1$ **(C)** $x = 1$ & $x = 4$

(D) $x = 1$ & $x = 3$ **(E)** None of these

Follow-up: *How would your answer change if you found the critical points for f, but the graph given was for f'?*

35. Suppose that the graph below is the graph of $f'(x)$, the derivative of a function $f(x)$. Find the open interval(s) where $f(x)$ is decreasing.



(A) $(-\infty, -2.2), (0, 2.2)$ **(B)** $(-2.2, 0), (2.2, \infty)$ **(C)** $(-\infty, -3), (-1, 1), (3, \infty)$

(D) $(-3, -1), (1, 3)$ **(E)** None of these

Follow-up: *How would your answer change if the graph was that of f?*

36. Find the critical numbers for the function

$$f(x) = 4x^3 - 3x^2 - 36x + 5.$$

Round to two decimal places as needed.

(A) $x = 4.00$ & $x = -3.00$ & $x = -36.00$ & $x = 5.00$ **(B)** $x = -1.50$ & $x = 2.00$

(C) $x = -2.72$ & $x = 0.14$ & $x = 3.34$ **(D)** There are no critical numbers

(E) None of these

37. Suppose the total cost $C(x)$ (in dollars) to manufacture a quantity x of weed killer (in hundreds of liters) is given by the function

$$C(x) = x^3 - 3x^2 + 6x + 60.$$

Where is $C(x)$ is increasing? Where is $C(x)$ is decreasing?

38. The cost (in dollars) of producing q headphones is given by

$$C(q) = 3q^2 - 3q + 48.$$

Identify the open interval where the average cost $\overline{C}(q)$ is increasing.

(A) $(0, 4)$ **(B)** $(0, \infty)$ **(C)** $(4, \infty)$ **(D)** There is no interval **(E)** None of these

39. A manufacturer sells video games with the following cost and revenue functions (in dollars), where x is the number of games sold.

$$C(x) = 0.17x^2 - 0.00012x^3$$

$$R(x) = 0.554x^2 - 0.0002x^3$$

Determine the interval(s) on which the profit function is increasing.

(A) $(0, 3200)$ **(B)** $(0, 4800)$ **(C)** $(1417, \infty)$ **(D)** $(0, \infty)$ **(E)** None of these

40. Assume that a demand equation is given by $p = 148 - q$. Identify the open interval(s) for $0 \leq q \leq 148$ where *revenue* is decreasing.

(A) $(0, 148)$ **(B)** $(0, 74)$ **(C)** $(74, 148)$ **(D)** There are no intervals **(E)** None of these

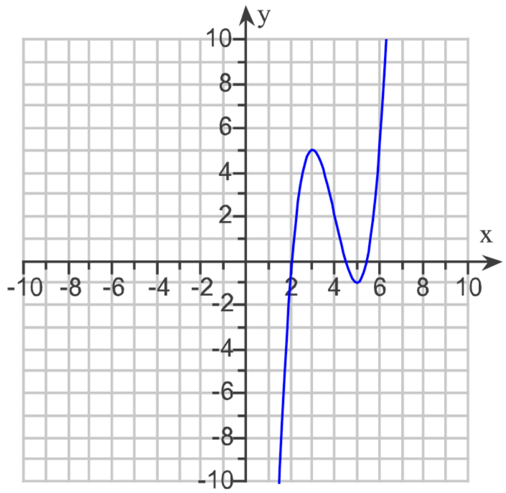
41. The projected year-end assets in a collection of trust funds, in trillions of dollars, where t represents the number of years since 2000, can be approximated by the following function where $0 \leq t \leq 50$.

$$A(t) = 0.0000284t^3 - 0.00450t^2 + 0.0682t + 4.89$$

Identify the open interval for $0 \leq t \leq 50$ where $A(t)$ is increasing. (Round to one decimal place as needed.)

(A) $(0, 8.2)$ **(B)** $(8.2, 50)$ **(C)** $(0, 50)$ **(D)** There is no interval **(E)** None of these

42. Find the location(s) of all relative maxima of $y = f(x)$ graphed below.

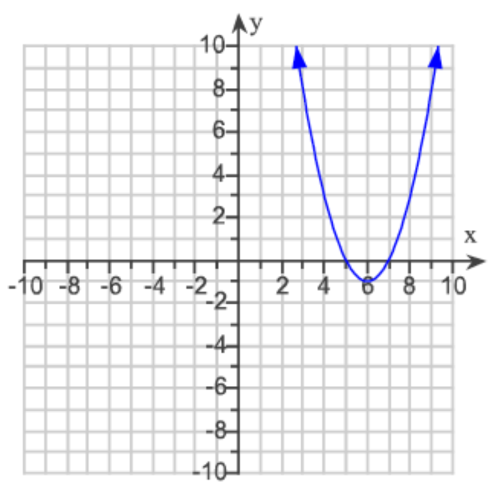


(A) $x = 2$ & $x = 5.3$ **(B)** $x = 4.5$ **(C)** $x = 5$

(D) $x = 3$ **(E)** None of these

FOLLOW-UP: *How would your answer change if you found the relative maxima for f, but the graph given was for f'?*

43. Suppose that the graph below is the graph of $f'(x)$, the derivative of a function $f(x)$. Find the location(s) of all relative extrema of $f(x)$, and tell whether each is a relative maximum or minimum.



(A) $x = 6$ is a relative maximum

(B) $x = 6$ is a relative minimum

(C) $x = 5$ is a relative minimum & $x = 7$ is a relative maximum

(D) $x = 5$ is a relative maximum & $x = 7$ is a relative minimum

(E) None of these

FOLLOW-UP: *How would your answer change if the graph was that of f?*

44. Find the locations of all relative extrema of $G(x) = -x^3 + 3x^2 + 24x + 5$, and tell whether each is a relative maximum or minimum.

(A) Relative minimums occur at $x = -3.5$ & $x = 6.7$,
and a relative maximum occurs at $x = -0.2$.

(B) A relative minimum occurs at $x = -0.2$,
and relative maximums occur at $x = -3.5$ & $x = 6.7$.

(C) A relative minimum occurs at $x = -2$,
and a relative maximum occurs at $x = 4$.

(D) A relative minimum occurs at $x = 4$,
and a relative maximum occurs at $x = -2$.

(E) None of these

45. Suppose $f(x)$ is a function whose derivative is given by

$$f'(x) = -2(x+3)(x-4).$$

Find the locations of all relative extrema of $f(x)$, and tell whether each is a relative maximum or minimum.

(A) A relative minimum occurs at $x = -3$ and a relative maximum occurs at $x = 4$

(B) A relative minimum occurs at $x = 4$ and a relative maximum occurs at $x = -3$

(C) A relative minimum occurs at $x = 0.5$

(D) A relative maximum occurs at $x = 0.5$

(E) None of these

46. Let $f(x)$ be a continuous function with two critical points whose derivative has the following values:

x	2	3	4	5	6
$f'(x)$	-2	0	1	0	-5

$f(x)$ has a relative maximum at

(A) $x = 2$ **(B)** $x = 3$ **(C)** $x = 4$ **(D)** $x = 5$ **(E)** None of these

47. A small company manufactures and sells lamps. The production manager has determined that the cost and demand functions for q lamps per week are

$$\text{Cost: } C(q) = 200 + 30qe^{-0.02q}; \text{ Demand: } p = 60e^{-0.02q}$$

where p is the price per lamp.

(i) Express the profit, P, as a function of q.

(ii) Find the number, q, of units that produces maximum profit. Why does this value of q produce maximum profit?

(iii) Find the price, p, per unit that produces maximum profit.

(iv) Find the maximum profit, P.

Formulas You Might Find Useful

$$
\begin{aligned}
I &= Prt \\
A &= P\left(1+\frac{r}{m}\right)^{mt} \\
A &= Pe^{rt} \\
r_E &= \left(1+\frac{r}{m}\right)^{m} - 1 \\
r_E &= e^r - 1 \\
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
\frac{d}{dx}\left[f(g(x))\right] &= f'(g(x))\cdot g'(x) \\
\frac{d}{dx}\left[u(x)\cdot v(x)\right] &= u(x)\cdot v'(x) + v(x)\cdot u'(x) \\
\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] &= \frac{v(x)\cdot u'(x) - u(x)\cdot v'(x)}{[v(x)]^2} \\
\frac{d}{dx}\left[a^x\right] &= (\ln a)a^x \\
\frac{d}{dx}\left[\log_a(x)\right] &= \frac{1}{(\ln a)x}
\end{aligned}
$$

Solutions to Study Guide - Part 2

1. C
2. A
3. D
4. C
5. B
6. D
7. B
8. **(i)** A
   (ii) A
9. A
10. D
11. C
12. B
13. A
14. **(i)** B
    (ii) D
    (iii) D
15. **(i)** B
    (ii) A
16. B
17. A
18. D
19. **(i)** A
    (ii) A
    (iii) B
    (iv) B
20. A
21. A
22. B
23. B
24. D
25. B
26. A

27. D

28. B

29. **(i)** C

    (ii) C

30. C

31. A

32. C

33. **(i)** A

    (ii) $MR(q) = 102 + \dfrac{54\ln(q) - 54}{(\ln(q))^2}$

    (iii) $P(q) = \dfrac{54q}{\ln(q)} - 91$

    (iv) E

34. D

35. D

36. B

37. $C(x)$ is always increasing; it is never decreasing

38. C

39. A

40. C

41. A

42. D

43. D

44. C

45. A

46. D

47. **(i)** $P(q) = 30qe^{-0.02q} - 200$

    (ii) $q = 50$

    (iii) $p = 22.07$

    (iv) $P = 351.82$